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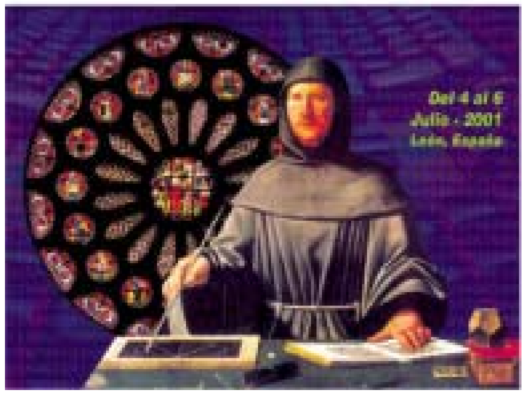

THE SOURCES OF HETEROGENEITY AMONG AGENTS AND VOLATILITY PERSISTENCE IN REAL RETURNS

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Abstract

This paper exemines the conditions for volatility pesistence in real returns in heterogeneous agent models with borrowing constraints, capital accumulation, production, and idiosyncratic endowment shocks. In contrast to previous studies, which did not include real capital accumulation and production, borrawing constraints are not necessary for the appearance of volatility persistence in real returus. However, volatility measures positively correlated with real returns and negatively correlated with gdp only when borrawing opportunity are restrictive.

Keywords: volatility persistence, hoterogenecus agents, parameterized expectations.

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1. Introduction

The significance of heterogeneous agent models for expleining key macroeconomic facts, has led to a call for a "requiem" for models based on the "representative agent". In particular, Carroll (2000) has drawn attention to heterogeneity in agents as a key ingredient for explaining the aggregate marginal propensity to consume as well as skewed wealth distribution. However, for Carroll, heterogeneity among consumers rests on different rates of time preference, not on differences in risk aversion. In the Carroll model, preferences are based on identical logarithmic utility functions among agents.

Recent research with heterogeneous agent models has focused on the presence of borrowing constraints to explain another key macroeconomic fact, namely, volatility clustering or persistence in real returns. Den Haan (1997, 1999), den Hann and Spear (1998), and Zhang (2000) all examine the issue of volatility persistence under different extensions of a stochastic endowment model with idiosyncratic shocks and borrowing constraints. In thaese models, risk aversion is identical among agents. In the experiments reported in these papers, the risk aversion coefficient for all the agents is set at values between .4 and 5.

In addition to heterogeneity in risk aversion among agents, there has been little analysis of the role of production opportunities in environments where borrowing/lending opportunities may be limited. Huggett (1997) introduced heterogeneous agents into a Brock-Mirman stochastic growth framework with idiosyncratic endowment shocks and a continuum of agents, but he did not consider the question of volatility persistence in real returns.

This paper uses a framework similar to those examined in recent literature but adds heterogeneity in risk aversion, as well as capital and production opportunities with limited lending/borrowing opportunities for individual agents. With these extensions, the following conclusions emerge with respect to "volatility persistence" in real returns and the presence or absence of borrowing limits:

- it does not matter if their are very tight binding borrawing limits or not

- it does not matter if there is "aggregate uncerteinty" or not;

- if the heterogeneous agents in the model are sufficiently "heterogeneous", in terms of differences in risk aversion, or different endowment processes, then real returns exhibit volatility persistence

- the results are robust for models with two or three agents-or more.

- borrawing constraints are more likely to predict strong negative correlations between time-varying risk and output as well as between timevarying risk and real returns.

2 .The Model

The usual constant relative risk aversion (CRRA) utility function characterizes the preferences of each agent or household:

$$U(c_t^i) = \frac{(c_t^i)^{1-\sigma_i}}{1-\sigma_i} \tag{1}$$

where σ_i is the coefflcient of relative risk aversion for agent i..

Each maximizes the following intertemporal discounted utility function over an infinite horizon:

$$E\left[\sum_{t=0}^{\infty} \beta^t U(c_t^i)\right] \tag{2}$$

with O <β < 1.

Each agent faces the following budget constraint:

$$c_t^i = r_t k_t^i + w_t e_t^i + (1-\delta)k_t^i - k_{t+1}^i \tag{3}$$

where e_t^i is the endowment of agent i at time t, k_t^i is the productive capital or bonds held by agent i at time t, and δ is the rate of depreciation.

The variebles w_t and r_t represent the real wages and return on capital at time t. There is a single firm that operates the technology, with marginal productivity conditions for wages and capital returns based on aggregate capital and labor, K and E.. Labor endowments follow a Markov chain, with no aggregate uncerteinty.

$$f(K, E) = AK^{\alpha}E^{1-\alpha} \tag{4}$$

$$w = f_E$$

$$r = f_K$$

$$K = \sum_{i=1}^{N} k^i$$

$$E = \sum_{i=1}^{N} e^{i} = 1$$

$$P_{kj} = \Pr\left(e_{t+1} = \overline{e_j} \middle| e_t = \overline{e_k}\right)$$

Each agent solves the dynamic discounted programming problem and faces a decision rule based on the following Euler equation:

$$(c_t^i)^{-\sigma_i} = \beta E\left[(c_{t+1}^i)^{-\sigma_i}(1 + r_{t+1} - \delta)\right]$$

3. Solution Method

To solve the Euler equation for the optimel decision rule for each agent, we make use of parameterized expectations, extensively analyzed by Marcet (1988, 1993), and den Haan and Marcot (1990).

The Euler equation is parameterized in the following way:

$$(c_t^i)^{-\sigma} = \beta \Psi^i (k_t^i, e_t^i, K_t; \gamma^i) \qquad (5)$$

where the functional form Ψ^i is a neural network, with arguments k_t^i, e_t^i, K_t and parameters γ^j.Each agent forms expectations on the basis of observing personal capital and labor endowments, as well as aggregate capital. Since there are only idiosyncratic shocks to endowments, there is no aggregate uncertainty with respect to the evolution of E.

The neural network specification of the expectations function $\Psi^i(k_t^i, e_t^i, K_t; \gamma^i)$ has the following form:

$$n_{l,t}^i = \sum_{j=1}^{J^*} b_j^i x_{j,t}^i \qquad (6)$$

$$N_{l,t}^i = \frac{1}{1 + e^{-n_{lo,t}^i}}$$

$$\hat{\Psi}_t^i = \sum_{l=1}^{L^*} k_l^i N_{l,t}^i$$

where J* is the number of exogenous or input variables, set at three, for $x_{j,t}^i = \left[1, k_t^i, e_t^i\right]$, K* is the number of neurons, n_t^i is a linear combination of the input variables, N_t^i is a logsigmoid or logistic transformation of n_t^i, and $\hat{\Psi}_t^i$ is the neural network prediction at time t of $E\left[(c_{t+1}^i)^{-\sigma_i}(1+r_{t+1}-\delta)\right]$ for agent i, summarized by the function $\Psi^i(k_t^i, e_t^i, K_t; \gamma^i)$, with the parameter set { γ^i } = $\left\{b_j^i, k_k^i\right\}$, j = 1,...,J*, l = 1,...,L*.

As seen in this equation, the only difference from ordinary nan-linear estimation relating "regressors" to a "regressand" is the use of the hidden nades or neurons, N. One forms a neuron by taking a linear combination of the regressors and then transforming this variable by the logistic or logsigmoid function. One then proceeds to thus one or more of thase neurons in a linear way to forecast the dependent variable $\hat{\psi}_t$.

Sargent (1997) has shown that the neural network specification does a better job of "approximating" any non-linear function than polynomial approximations, in that sense that a neural network achieves the same degree of in-sample predictive accuracy with fewer parameters than a polynomial approximation, or achieves greater accuracy than a polynomial one, using the same number of parameters.

The main choices that one has to make for a neural network is L*, the number of hidden neurons, for predicting a given variable Ψ^i. Generally, a neural network with only one hidden neuron closely approximates a simple linear model, whereas larger numbers of neurons approximate more complex non-linear relationships. Obviously, with a larger number of neurons in the hidden layer of the network, one may approximate progressively more complex non-linear phenomena, but at the cost of an increasingly larger parameter set.

The approach of this study is to use relatively simple neural networks, between two and four neurons, in order to show that even relatively simple neural network specifications do well for approximating non-linear relations implied by forward-looking expectations in stochastic dynamic general equilibrium models.

Each agent solves the optimization problem for γ^i in order to minimize the sum of squares of the following error metric:

$$\underset{\gamma^i}{Min} \sum_{t=1}^{T} \left[\varsigma_t^i\right]^2 \tag{7}$$

$$\varsigma_t^i = \beta\Psi_t^i(k_t^i, e_t^i, K; \gamma^i) - \beta E\left[(c_{t+1}^i)^{-\sigma_i}(1+r_{t+1}-\delta)\right] \tag{8}$$

The error function is minimized, subject to the following constraints:

$$K_t > 0 \tag{9}$$

$$c_t^i > 0$$

(10)

$$b_t \leq \lambda K_t$$

(11)

$$b_t = \sum_i \left| k_t^i \right| - K_t$$

(12)

where b_t represents aggregate borrowing at time t. Individuals are net borrawers if their asset holding are less than zero.

Since the parameterized expectation solution is a relatively complex nonlinear function, the optimization problem is solved with a repeated hybrid approach. First a global search method, genetic algorithm, similar to the one develaped by Duffy and McNelis (2000), is used to find the initial parameter set $\{ \gamma^i \}$, then a local optimization, the BFGS method, based on the quasi-Newton algorithm, is used to "fine tune" the genetic algorithm solution.

De Faleo (1998) applied the genetic algorithm to nonlinear neural network estimation, and found that his results "proved the effectiveness" of such algorithms for neural network estimation.. The main drawback of the genetic algorithm is that it is slow. For even a reasonable size or dimension of the coefficient vector, the various combinations and permutations of the coefficients which the genetic search may find "optimal" or close to optimal, at various generations, may become very large. This is another example of the well-known "curse of dimensionality" in non-linear optimization. Thns, one neads to let the genetic algorithm "run" over a large number of generations—perhaps several hundred—in order to arrive at results which resemble unique and global minimum points.

Quagliarella and Vicini (1998) point out that hybridization may lead to better solutions than those obtainable using the two methods individually. They argue that it is not necessary to carry out the quasi-Newton optimization until convergence, if one is going to repeat the process several times.. The utility of the quasi-Newton BFGS algorithm is its ability to improve the "individuals it treats", so "its beneficial effects can be obteined just pertorming a few iterations each time" [Quagliarella and Vicini (1998), p. 307].

In this study, the number of agents varies from two to three, while the borrowing restrietion is progressively relexed in a series of alternative experiments. The nature of the endowment processes also varies, in order to see how sensitive the presence or absence of GARCH in real returns may be to alternative specifications.

4. Calibration

Table I lists the parameter configuration we use in the ba£eline simulations of the model.

Table I: Parameter Specification

Discount Rate	β = .96
Production Fn. and Depreciation	$A = 1$, α = .36, δ = .1
Borrawing Limits	λ = { 1, 25, .5,1, 1.5, 3}
Risk Aversion-2 agents	σ = {.5, 3}
Risk Aversion-3 agents	σ = {.5,1.5, 3}
Endowments-2 agents	e^1=1/2[.8, 1.2],$\pi(e_j^1 \| e_j^1)$ = .5
	$\pi(e_k^1 \| e_j^1) = .5, e^2 = 1 - e^1$
Endowments-3 agents	e^1=1/3[.8, 1.2],$\pi(e_j^i \| e_j^i)$ = .5

$$\pi(e_k^i | e_j^i) = .5, i = 1,2; e^3 = 1 - e^1 - e^2$$

The parameter specification is similar to previous studies. The ranges for the idiosyncratic shocks are slightly smeller than those used by den Hann (1997), who calibrated his model for monthly data, but identical to those used by Huggett (1997). The transition probabilities are identical to Huggett's specification, as are the production function coefficients the rate of depreciation, and the discount rate.

The sample size for the model is 2000. The number of neurons set for each agent is two. Each agent knows its own current endowment shock and lagged capital stock, as well as the lagged capital and lagged endowment shocks of the other agents. Thus the neural network parameterized expectation approximation for each is a function of four "state variables" in the two-agent model, and a function of six state variebles in the three-agent model.

5. Simulation Results

This section first takes up the results for the two agent model, and then discusses the three-agent model.

5.1 Two Agent Model

The key results from simulations of the two agent model appear in Table 2.

Table 2
Summary Statistics: Two Agent Model
Borrowing Restriction (% K)

	0.25	*0.5*	*1*
Mean Borrowing	0.603917	2.255711	2.542171
Consumption:1	0.516968	0.442467	0.431263
Consumption:2	0.793493	0.844199	0.864836
Real Returns			
Standard Deviation	0.004228	0.003769	0.004731
Skewness	- 1.009439	-0.554032	0.456481
Kurtosis	5.733738	4.139428	3.275868
GARCH Coeff	0.625015	0.807888	0.668315
ARCH Coeff	0.374983	0.127726	0.2311 82
Correlation: cond variance, real returns	0.287599	0.050931	-0.00695
Correlation: cond variance, gdp	-0.295732	-0.052837	0.001739

Table 2 shows, as expected, that mean borrowing increases as restrictions on borrowing or lending are relaxed. Similarly, consumption of agent two, with a higher relative risk aversion caefficient, becomes higher as the restitutions on borrowing are relaxed. However there is no discernible pattern in the behavior of the standard deviation as λ increases. However, the skewness of real returus increases as financial restrictions are relaxed, as λ increases from .25 to 1, while kurtosis decreases.

However, Table 2 aleo shows that the GARCH or ARCH coefficients are present and significant for real returns, whether the restrictions on borrowing and lending are tight or loose.

The correlations between the conditional variance of the returns and the level of returns are positive, under relatively tight restrictions, but decline and become negligible as λ approaches 1. Similarly the correlations between the conditional variance and gdp growth are negative under tight restrictions but also decline in absolute value.

Similar values for these correlation coefficients are reported by den Haan and Spear [(1998), p. 449] for U.S. data. The reported correlations between the conditional variance and real returns fell in a range of [.414, .415], while the correlations botween the conditional variance and gdp growth fell in a range of [-.17, -.21].

Figure 1 pictures the cross correlations between the conditional variance the level of real returns for the alternative borrowing restrictions.

Figure 1

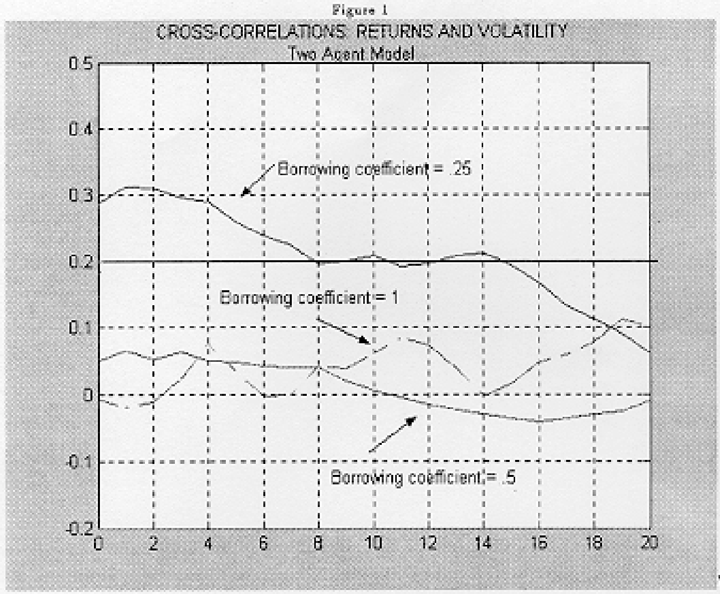


Figure 1 represents the cross correlations between the conditional variance and real returns, for lags 0 through 20. The upper bound for significance is .20 and the lower bound is -.20. Figure 1 shows that the cross correlation between real returns and its own volatility is significant for lags zero through eight, for the case of $\lambda = .25$.

Figure 2 pictures the cross correlations for the conditional variance and gdp.

Figure 2

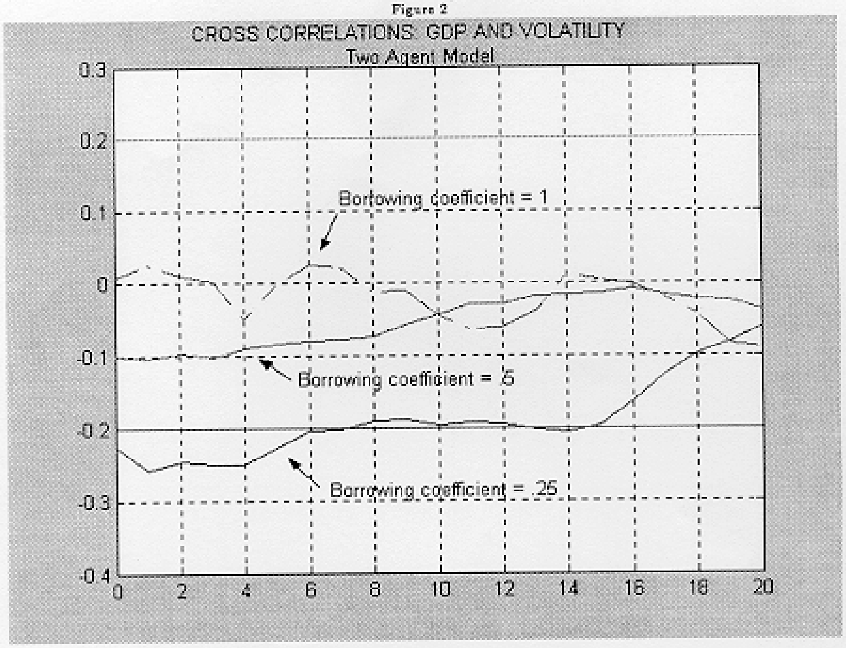


Figure 2 follows a similar pattern as Figure 1. The lower and and upper bounds for significance are [-.2, 2]. With respect to the cross correlations between volatility and real gdp, only for the case of $\lambda = .25$ are the cross correlations "significant" and negative, for approximately six periods. For higher values of λ, the cross correlations are not significant.

5.2 Three Agent Model

The results from the three agent model appear in Table 3.

Table 3
Summary Statistics: Three Agent Model

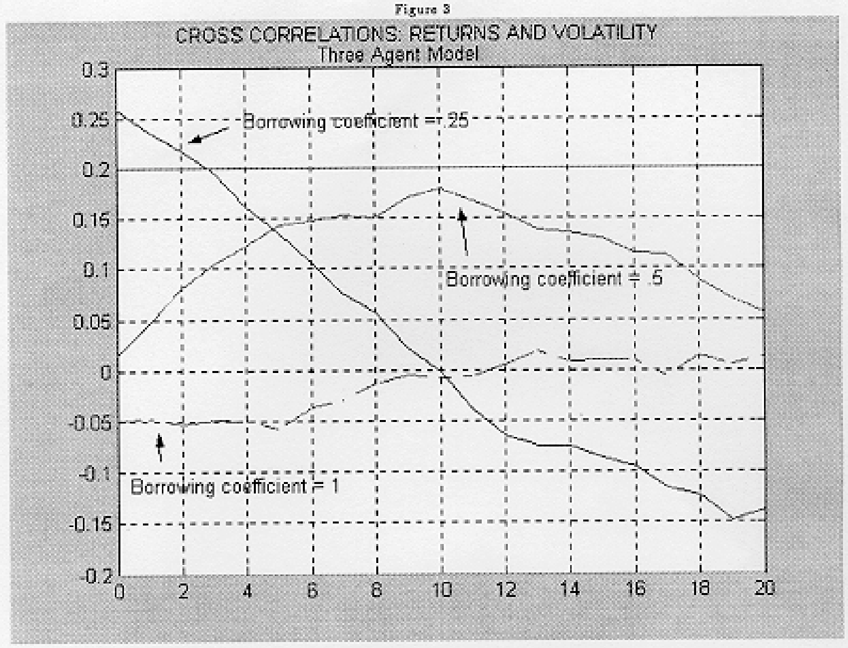
Borrowing Restriction (% K)	*0.25*	*0.5*	*1*
Mean Borrowing	1.210354	2.978925	5.943099
Consumption:1	0.184746	0.236955	0.201595
Consumption:2	0.499165	0.469993	0.518401
Consumption:3	0.60204	0.622154	0.639938
Real Returns			
Standard Deviation	0.008094	0.008962	0.008745
Skewness	-1.636504	-1.822565	-1.803509
Kurtosis	4.467825	5.1026	5.705825
GARCH Coeff	0.544928	0.103151	0.561453
ARCH Coeff	0.160656	0.735579	0.214074
Correlation: cond variance, real returns	0.2581	0.016	-0.0492
Correlation: cond variance, gdp	-0.226807	-0.051801	0.148807

In the three agent model, as in the two agent model, one notes that total borrowing progressively increases as the restrictions are relaxed. With the constant relative risk aversion coefficients set at {.5, 1.5, 3}, one notes that the mean consumption ranking, of the three agents, ramains the same, whatever the value of λ, with more risk averse agents consuming more than the less risk averse agents.

As in the two agent model, the GARCH and ARCH coefficients are positive and significant for all values of λ. Similarly, for λ = .25 the tabulated correlations of the conditional variance with real returns and with gdp growth are reasonably close matches with those reported by den Haan and Spear (1998) for the US.

Figure 3 pictures the cross correlations for the level of real returns and its own lagged conditional variances for the three agent model.

Figure 3



The figures closely match those given in Figure 1 for the two agent model. The cross correlations of real returns with lagged volatility measures are significant only for the case of $\lambda = .25$, and are significant only for three periods.

Figure 4 pictures the cross correlations between the conditional variance and gdp growth, for the three agent model.

Figure 4

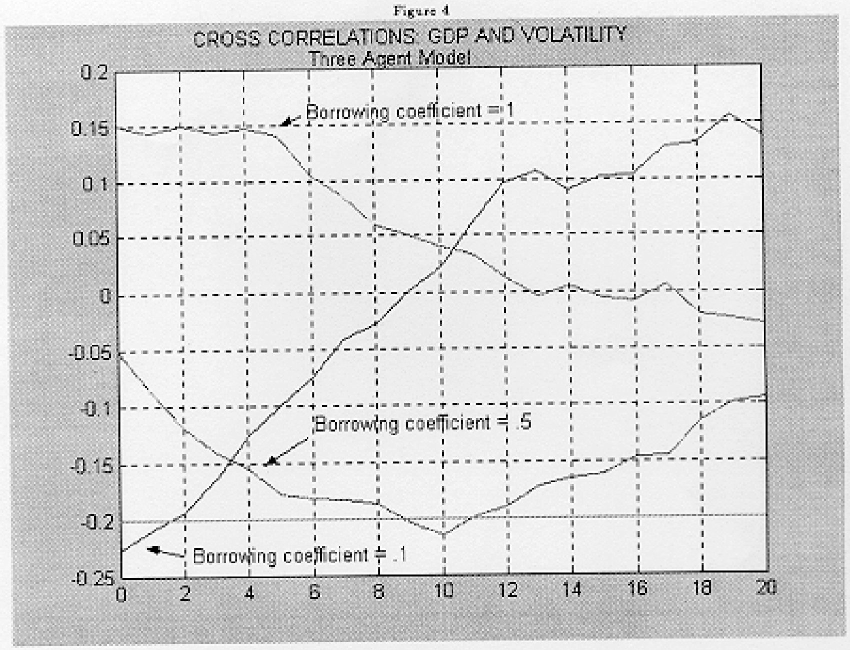


Figure 4 is quite similar to Figure 3. Only for the case of $\lambda = .25$, are the cross correlations "significantly" negative, and only for at most two lags.

6. Conclusion

This paper chellenges a conclusion of den Haan and Spear, that "asset returns are more volatile whenever financial frictions restrict the amount that agents want to trade" [den Haan and Spear (1998), p. 451]. Significant GARCH or ARCH parameters appear in real asset returns, once production is introducted, whether restrictions on borrowing are relatively tight or relatively slack. What is crucial is that agents be relatively heterogeneous.

This does not mean that the volatility of asset returns has the same effects in all environments. Depending on whether there are tight or loose restrictions, the volatility of the asset returns may have positive or negative effects on returns as well as on productivity. The simulations provide some evidence that in cases when borrowing limits are less than 25 percent of the level of total productive capital, then real return volatility matters much more. It matters much more, in these cases, simply because such real volatility is associated with higher real asset returns and lower gdp growth.

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